Princess Sumaya University for Technology
King Abdullah II School of Engineering

EE27355
Communication Principles

Quiz #3
Tuesday 24/3/2026

Name:………………………………………………………………. Section 1

Q.1)

(a) Find the Compact Fourier Series and sketch the amplitude and phase spectra for w(t) [see Figure Q.1], where the periodicity is equal 10π. Please take the integration limits from -5π to 5π. [14-Points]

(b) From the result that you got in part (a), find the exponential Fourier Series and sketch the amplitude and phase spectra for w(t). [6-Points]

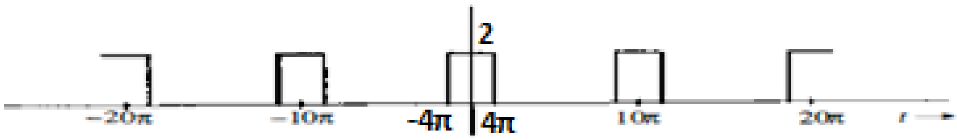


Figure Q.1

<u>Solution:</u> [20-Points]

$$a_0 = \frac{1}{T_0} \int_{T_0} g(t)\,dt$$

$$= \frac{1}{10\pi} \int_{-5\pi}^{5\pi} g(t)\,dt$$

$$= \frac{1}{10\pi} \int_{-4\pi}^{4\pi} 2\,dt$$

$$= \frac{2}{10\pi} \left(4\pi - (-4\pi)\right)$$

$$= \frac{8}{5}$$

$$a_n = \frac{2}{T_0} \int_{T_0} g(t) \cos n w_0 t\,dt$$

$$w_0 = \frac{2\pi}{T_0} = \frac{2\pi}{10\pi} = \frac{1}{5}$$

$$a_n = \frac{2}{10\pi} \int_{-4\pi}^{4\pi} (2) \cos \frac{n}{5} t\,dt$$

$$= \frac{2}{5\pi}\ \frac{1}{n}(5)\ \sin \frac{n}{5} t \Big|_{-4\pi}^{4\pi}$$

$$= \frac{2}{n\pi} \left[\sin \frac{n}{5} 4\pi - \sin -4\frac{n\pi}{5} \right]$$

$$= \frac{4}{n\pi} \sin \frac{n}{5} 4\pi$$

$$a_n = \begin{cases} \frac{4}{\pi}\sin 4\frac{\pi}{5} & n=1 \quad \text{+ve} \\ \frac{4}{2\pi}\sin 4\frac{2\pi}{5} & n=2 \quad \text{-ve} \\ \frac{4}{3\pi}\sin 4\frac{3\pi}{5} & n=3 \quad \text{+ve} \\ \frac{4}{4\pi}\sin 4\frac{4\pi}{5} & n=4 \quad \text{-ve} \\ \varnothing & n=5 \\ \frac{4}{6\pi}\sin 4\frac{6}{5}\pi & n=6 \quad \text{+ve} \\ \frac{4}{7\pi}\sin 4\frac{7}{5}\pi & n=7 \quad \text{-ve} \\ \frac{4}{8\pi}\sin 4\frac{8}{5}\pi & n=8 \quad \text{+ve} \\ \frac{4}{9\pi}\sin 4\frac{9}{5}\pi & n=9 \quad \text{-ve} \\ \varnothing & n=10 \end{cases}$$

$$b_n = \varnothing$$

$$g(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos n\omega_0 t + \cancel{b_n}\sin n\omega_0 t\right)$$

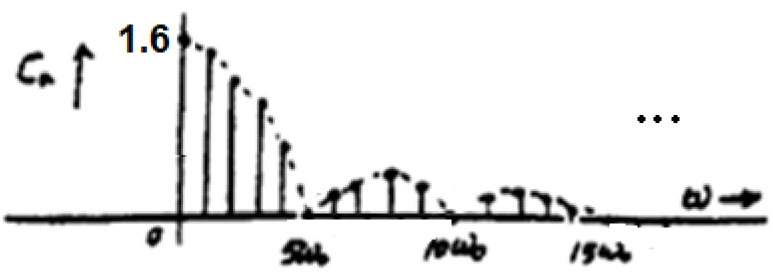
$$= \frac{8}{5} + \frac{2}{\pi}\Big[\sin 4\frac{\pi}{5}\cos\frac{1}{5}t + \frac{1}{2}\overset{\text{-ve}}{\sin 4\frac{2\pi}{5}}\cos\frac{2}{5}t + \frac{1}{3}\sin 4\frac{3\pi}{5}\cos\frac{3}{5}t + \frac{1}{4}\overset{\text{-ve}}{\sin 4\frac{4\pi}{5}}\cos\frac{4}{5}t + \varnothing$$

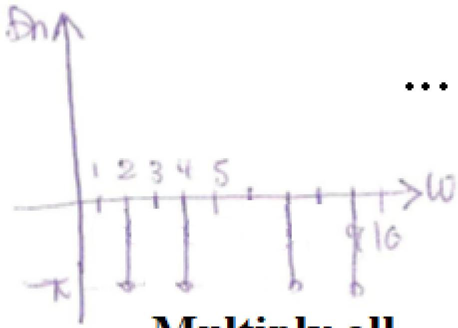
$$+ \frac{1}{6}\sin 4\frac{6\pi}{5}\cos\frac{6}{5}t + \frac{1}{7}\overset{\text{-ve}}{\sin 4\frac{7\pi}{5}}\cos\frac{7}{5}t + \frac{1}{8}\sin 4\frac{8\pi}{5}\cos\frac{8}{5}t + \frac{1}{9}\overset{\text{-ve}}{\sin 4\frac{9\pi}{5}}\cos\frac{9}{5}t + \varnothing + \cdots\Big]$$

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n\cos(n\omega_0 t + \theta_n)$$

$$= \frac{8}{5} + \frac{4}{\pi}\Big[\sin 4\frac{\pi}{5}\cos\frac{1}{5}t + \frac{1}{2}\sin 4\frac{2\pi}{5}\left(\cos\frac{2}{5}t - \pi\right) + \frac{1}{3}\sin 4\frac{3\pi}{5}\cos\frac{3}{5}t + \frac{1}{4}\sin 4\frac{4\pi}{5}\cos\left(\frac{4}{5}t - \pi\right) + \varnothing$$

$$+ C_6\cos\left(\frac{6}{5}t\right) + C_7\cos\left(\frac{7}{5}t - \pi\right) + C_8\cos\left(\frac{8}{5}t\right) + C_9\cos\left(\frac{9}{5}t - \pi\right) + \varnothing + \cdots\Big]$$





Multiply all numbers by w_0

$$w(t) = D_0 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} D_n e^{jnw_0 t}$$

$$D_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t)\,dt$$

$$= \frac{1}{10\pi} \int_{-5\pi}^{5\pi} w(t)\,dt$$

$$= \frac{1}{10\pi} \int_{-4\pi}^{4\pi} 2\,dt$$

$$= \frac{2}{10\pi} \left[4\pi - (-4\pi)\right]$$

$$= \frac{8}{5}$$

$$D_n = \frac{1}{T_0}\int_{T_0} w(t)\, e^{-jnw_0 t}\, dt$$

$$W_0 = \frac{2\pi}{T} = \frac{2\pi}{10\pi} = \frac{1}{5}$$

$$= \frac{1}{10\pi}\int_{-4\pi}^{4\pi} e^{-jnw_0 t}\, dt$$

$$= \frac{1}{10\pi}\int_{-4\pi}^{4\pi} 2\, e^{-j\frac{n}{5}t}\, dt$$

$$= \frac{2}{10\pi}\,\frac{-1}{j\frac{n}{5}}\, e^{-j\frac{n}{5}t}\Big|_{-4\pi}^{4\pi}$$

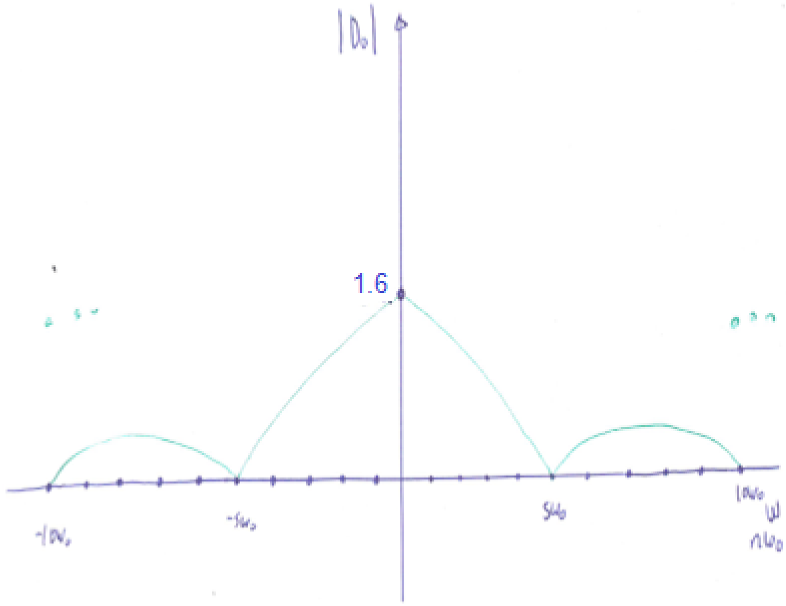
$$= \frac{-1}{j2n\pi}\left[e^{-j\frac{n}{5}4\pi} - e^{j\frac{n}{5}4\pi}\right]$$

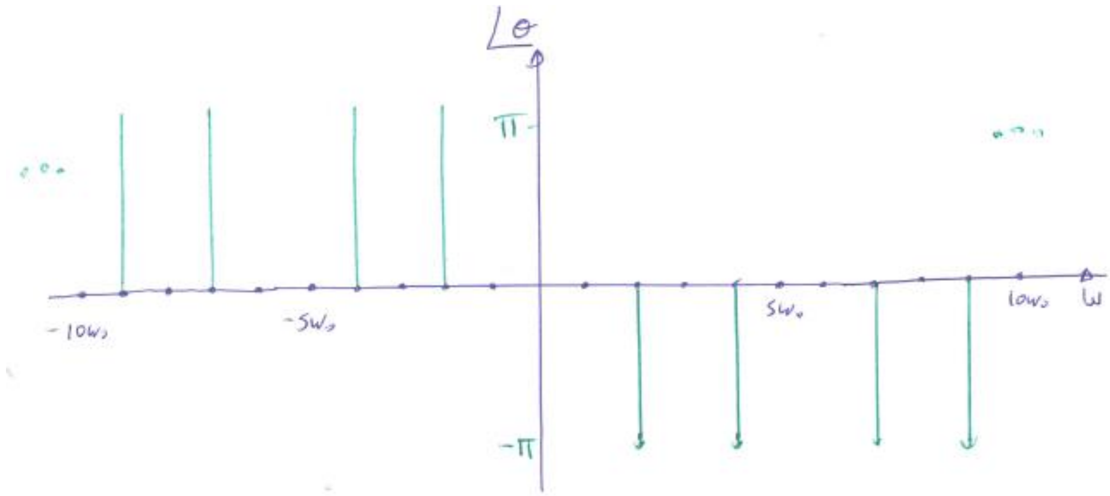
$$= \frac{1}{n\pi}\,\frac{e^{j\frac{n}{5}4\pi} - e^{-j\frac{n}{5}4\pi}}{2j}$$

$$= \frac{2}{n\pi}\sin\left(\frac{n}{5}4\pi\right)$$

$$w(t) = D_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} D_n\, e^{jnw_0 t}$$

$$= \frac{8}{5} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{2}{n\pi}\sin\frac{n}{5}4\pi\; e^{jnw_0 t}$$





Hint:

Trignometric Fourier Series:

$$a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t)\, dt$$

$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \cos n\omega_0 t\, dt \qquad n = 1, 2, 3, \ldots$$

$$w(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$b_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \sin n\omega_0 t\, dt \qquad n = 1, 2, 3, \ldots$$

Compact Fourier Series:

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n) \qquad t_1 \le t \le t_1 + T_0$$

$$C_0 = a_0 \qquad C_n = \sqrt{{a_n}^2 + {b_n}^2} \qquad \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

Exponential Fourier Series:

$$g(t) = D_0 + \sum_{n=1}^{\infty} D_n e^{jn\omega_0 t} + D_{-n} e^{-jn\omega_0 t} = D_0 + \sum_{n=-\infty\ (n\neq 0)}^{\infty} D_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$D_0 = a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t)\, dt \qquad D_n = \frac{1}{T_0} \int_{T_0} g(t) e^{-jn\omega_0 t}\, dt$$

$$\int e^{cx}\, dx = \frac{1}{c} e^{cx} \qquad \int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c \qquad \int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\sin x = \operatorname{Im}\{e^{ix}\} = \frac{e^{ix} - e^{-ix}}{2i} \qquad \cos x = \operatorname{Re}\{e^{ix}\} = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin^2(t) + \cos^2(t) = 1 \qquad \sin(-t) = -\sin(t) \qquad \cos(-t) = \cos(t)$$

$$\sin^2(x) = \tfrac{1}{2}[1 - \cos(2x)] \quad \cos^2(x) = \tfrac{1}{2}[1 + \cos(2x)]$$

$$\sin(x) = \cos(90° - x) \qquad \cos(x) = \sin(90° - x)$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$